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Ensuring an Adequate Safe Supply of Critical Minerals and Strategic Materials: An Optimization Approach for Supply Chain Modeling

Dominic Flocco

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About This Publication

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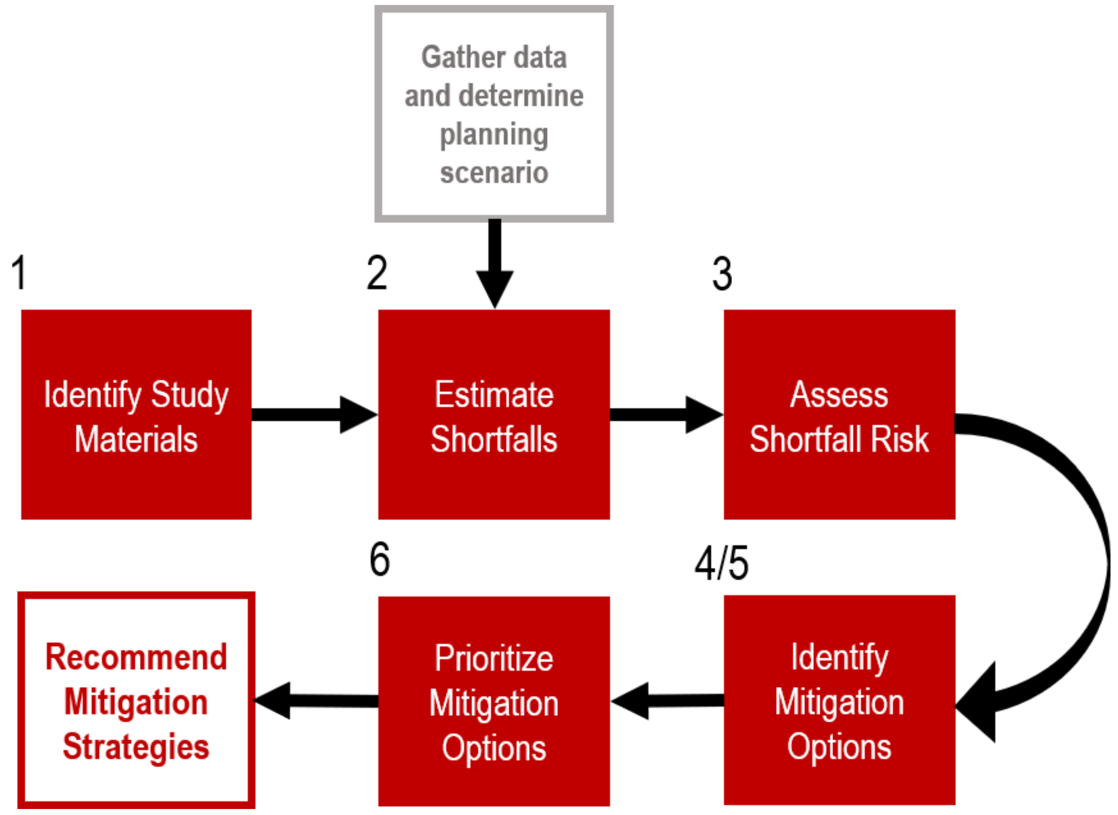
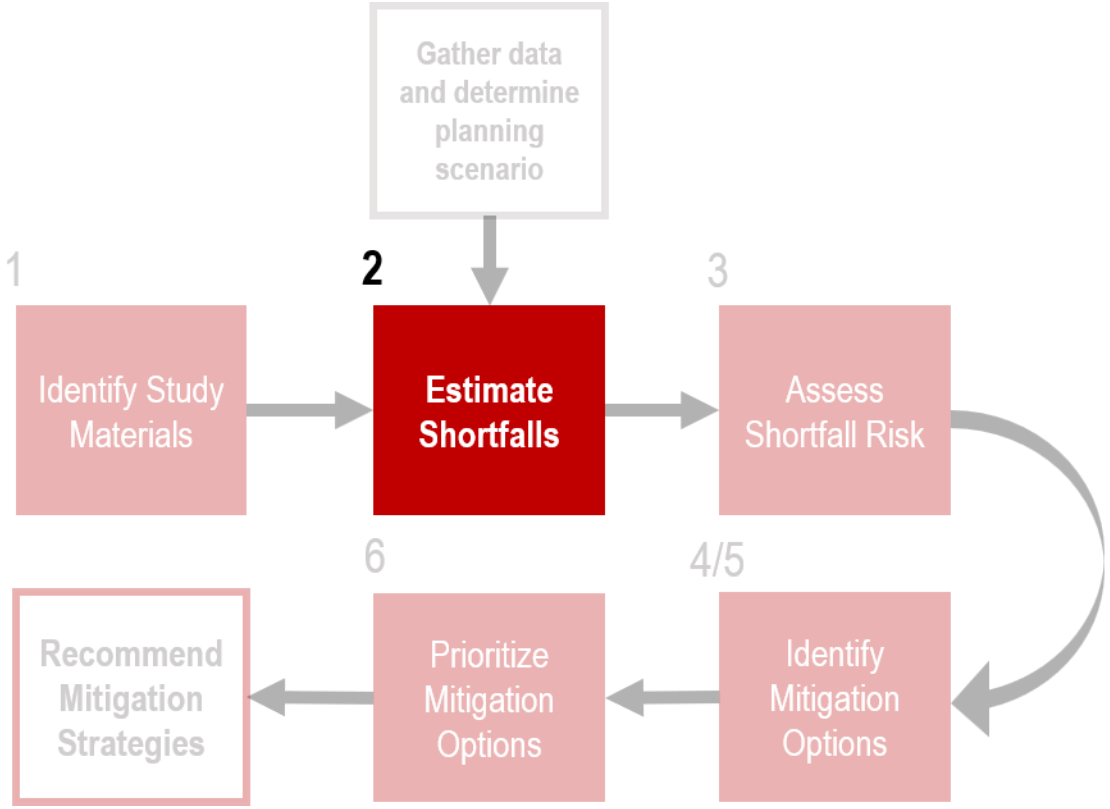
Background: National Defense Stockpile (NDS) Program

- DLA Strategic Materials operates and maintains a stockpile of **strategic and critical materials**; the Office of the Secretary of Defense manages the program
- Stockpiled material may be used to produce goods and services required for essential civilian and defense needs
- IDA supports DoD by helping to estimate and identify essential demands for S&CMs, safe supplies, gaps, and priorities for filling any gaps
- For this work, IDA uses a framework called “RAMF-SM” (Risk Assessment and Mitigation Framework for Strategic Materials)
- RAMF-SM informs:
  - **Biennial reports to Congress** on the National Defense Stockpile (1990s–present)
  - **Prioritized investments** for strategic materials based on an ROI strategy
  - “Deep dive” assessments and **supply chain mapping** of key strategic materials or components
  - Business case analyses identifying **risk mitigation strategies** beyond stockpiling (e.g., expanding domestic capacity, qualifying new suppliers)

RAMF-SM Steps

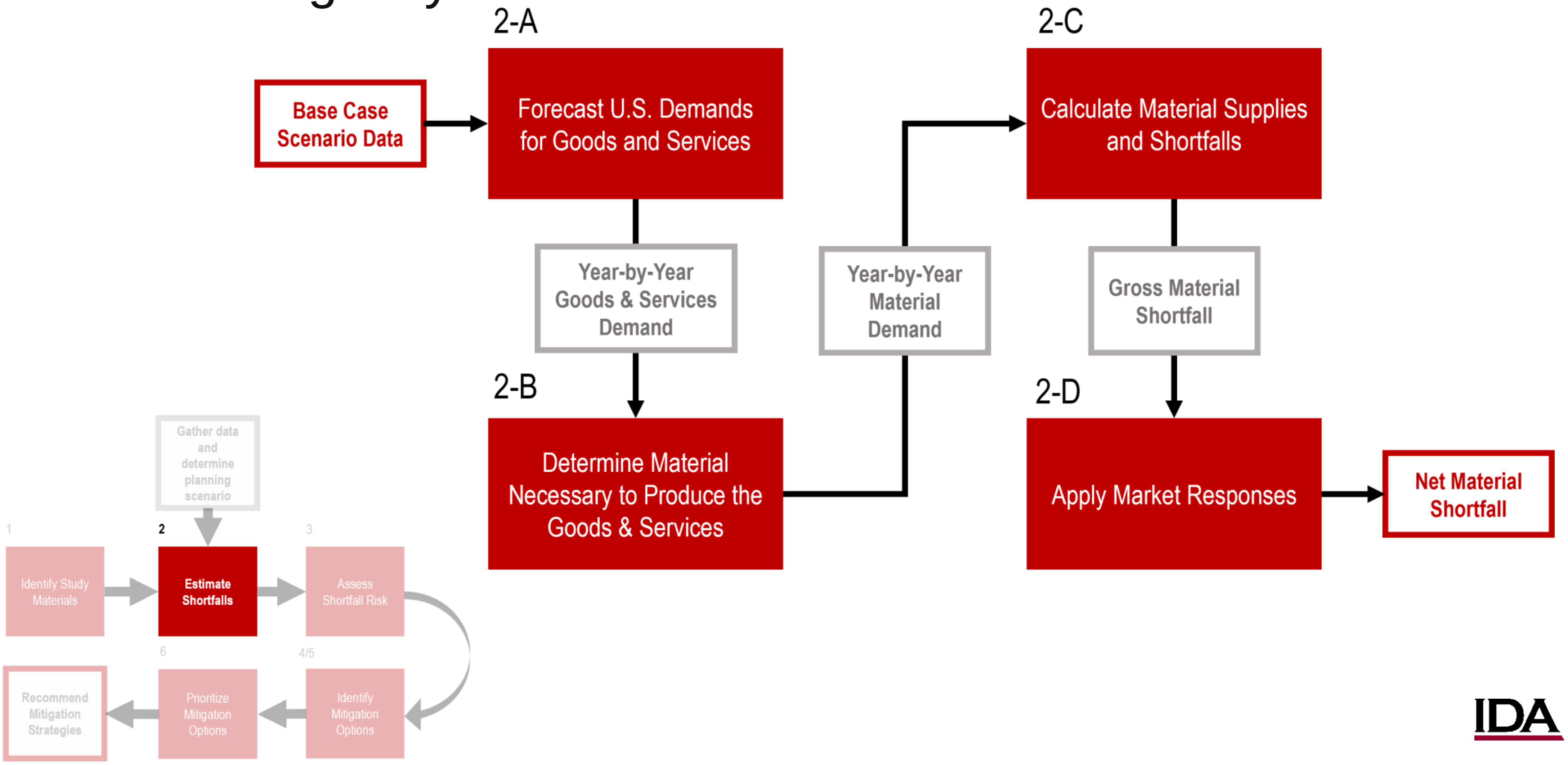


RAMF-SM Steps



Step 2 of RAMF-SM

Question: What U.S. material shortfalls would occur during a national emergency?



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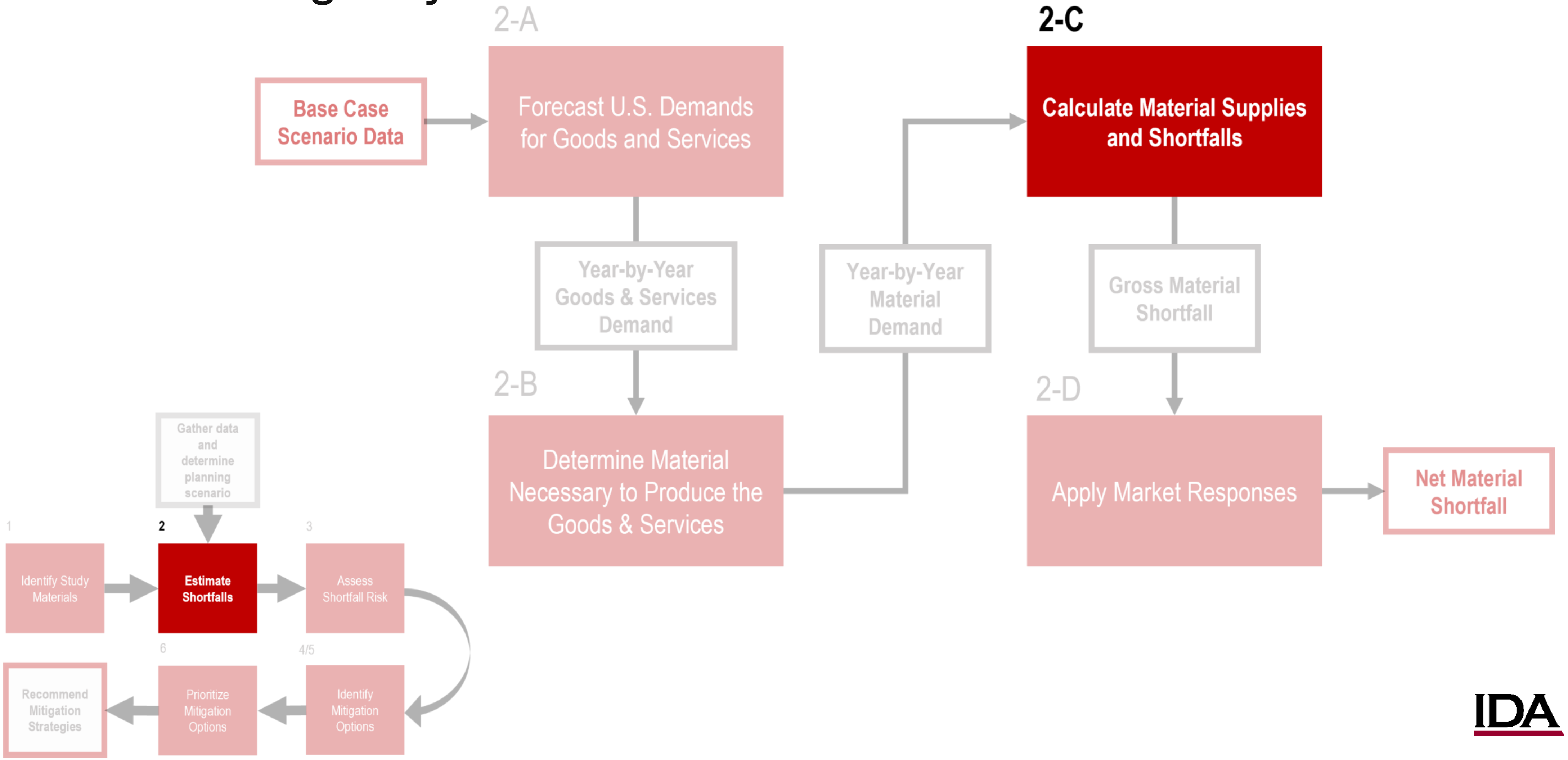
Question: What U.S. material shortfalls would occur during a national emergency?



Motivation for Optimization-Based Modeling

- Global supply chains of mined materials are complex, and material flow is susceptible to various decrement factors
- Material is demanded in manufacture-ready form (e.g., metal wire, sheet, rod), but supply is often examined in a different form (e.g., ore)
- While demand for finished metal is related to demand for ore, explicitly tracing the stages of production may lead to new insights
  - Material loss or supply bottlenecks may occur at any stage of production

Goal: Develop a general mathematical framework that models the downstream flow of material production through the global supply chain.

Two-Stage Production Model

- Consider a simple two-stage production process (e.g., mining and refining) of a single material over a four-year planning scenario
- Given the production capacities of each producer, is there a feasible route of material flow to meet U.S. demand?
  - If not, what is the shortfall? And where is the bottleneck in production?

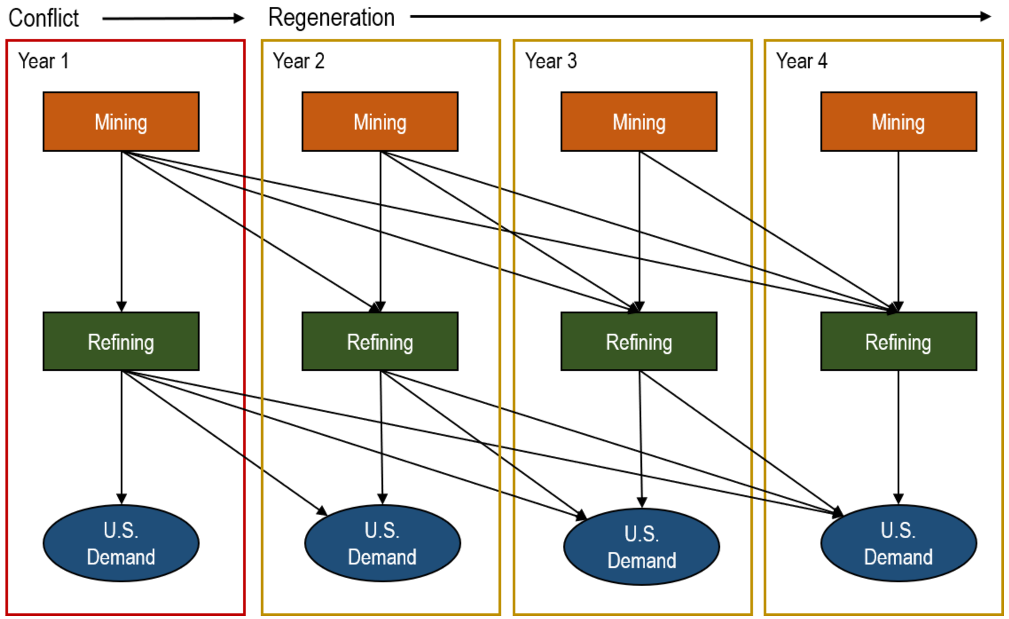


Figure. Time-space network representation of raw material supply chain.

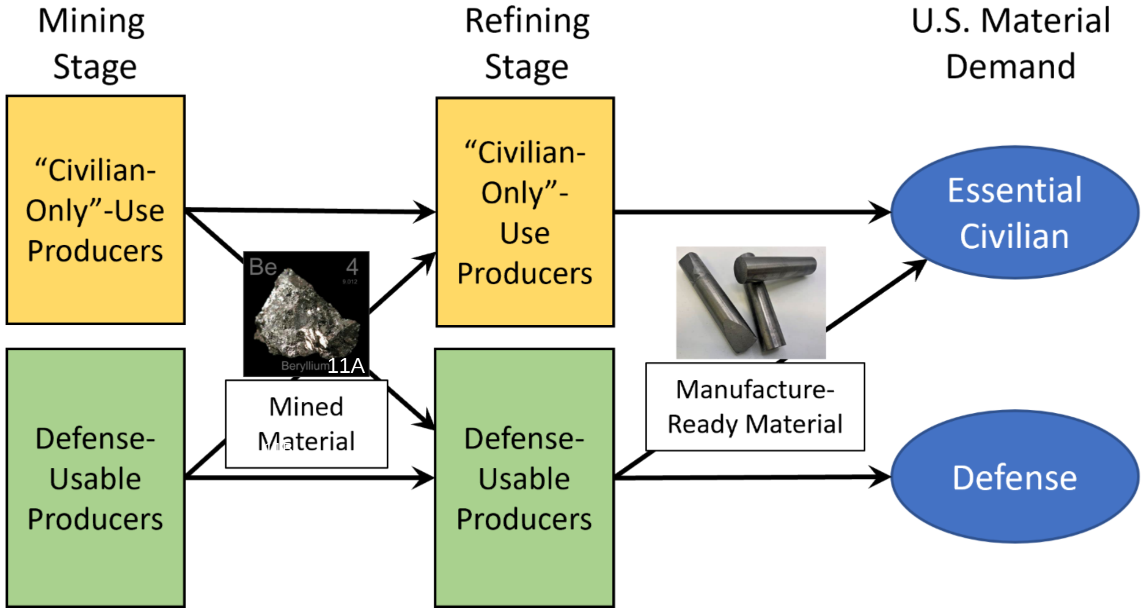
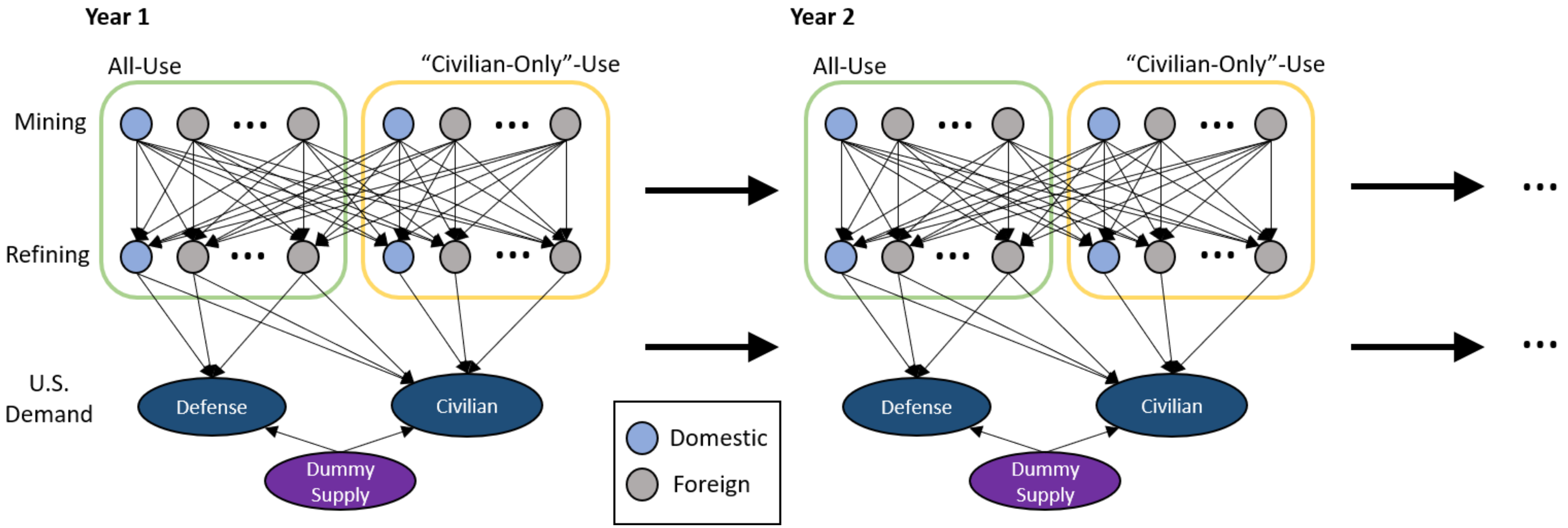


Figure. Usability classifications and flow restrictions.

Multi-Commodity Network Flow Approach

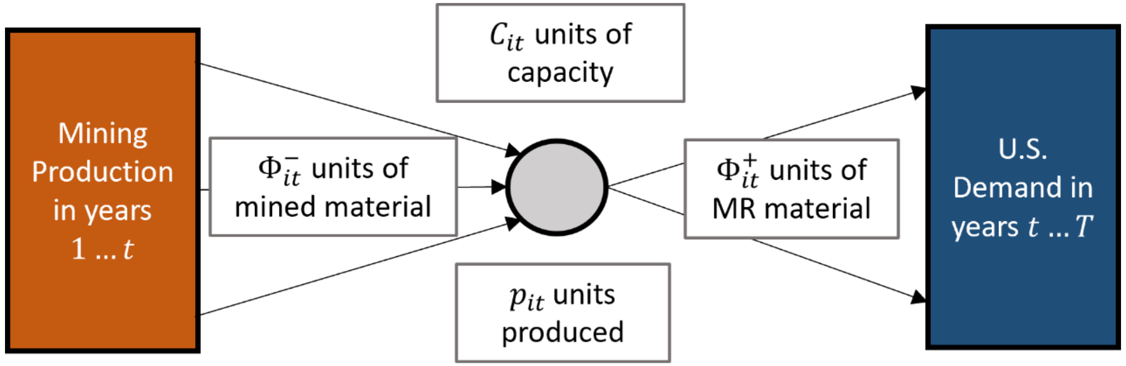
1. Create a **directed graph** to represent the global supply chain
2. Formulate **flow constraints** and incorporate **decrements to supply**
3. Construct an **optimal flow** on the network (i.e., numerical values on each edge) via linear programming



Production and Feedstock Constraints

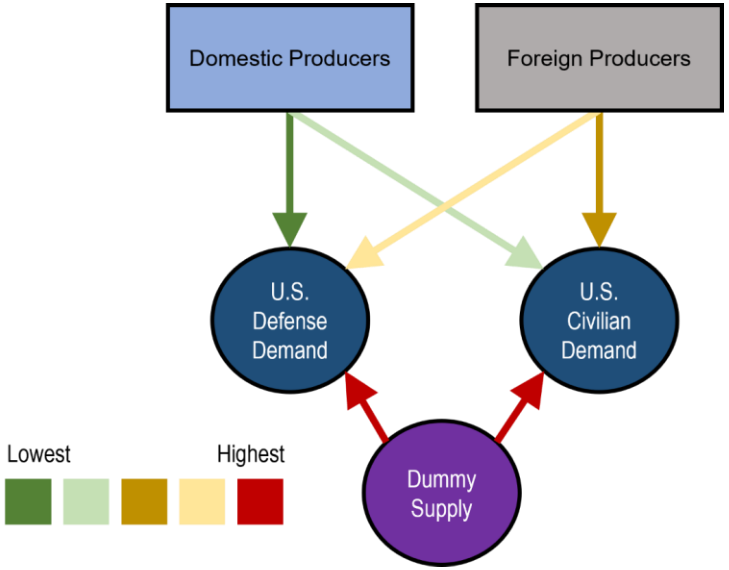
- Let C_{it} denote the production capacity of refinery i in year t
  - Decremented by producer's ability factor
- Let Φ_{it}^{-} denote the amount of mined material feedstock arriving at refinery i in year t
  - Decremented by shipping loss factor
- Let Φ_{it}^{+} denote the amount of manufacture-ready (MR) material leaving refinery i in year t
  - Decremented by material wastage factor
- Refining production p_{it} is limited by **both** refinery capacity and arriving feedstock
  - For refinery i in year t, we must have

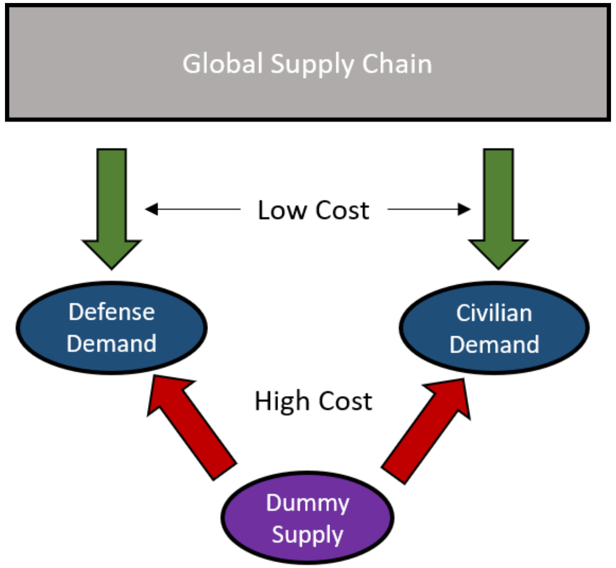
$$p_{it} \leq \Phi_{it}^{-} \quad \text{and} \quad p_{it} \leq C_{it}$$



Objective Function and Arc Costs

- **Objective:** Minimize the total cost of material flow across the network
- Arc costs are applied to each directed edge to **prioritize** or **restrict** certain material flows
- The **dummy supply node** is used to model material shortfall
  - It contains an unlimited amount of supply, but the cost of flow is set to be high relative to other sources of material flow
- The optimal solution will have positive flow from the dummy supply node *only* if there is no other feasible flow pattern to satisfy demand
  - Demand satisfaction is enforced through LP equality constraints





General LP Formulation

At a high level, the LP formulation of the downstream material production model is formulated as:

$$\text{Minimize:} \qquad \left. \textit{Total Cost of Material Flow} \right\} \mathbf{c}^{\top}\mathbf{x}$$

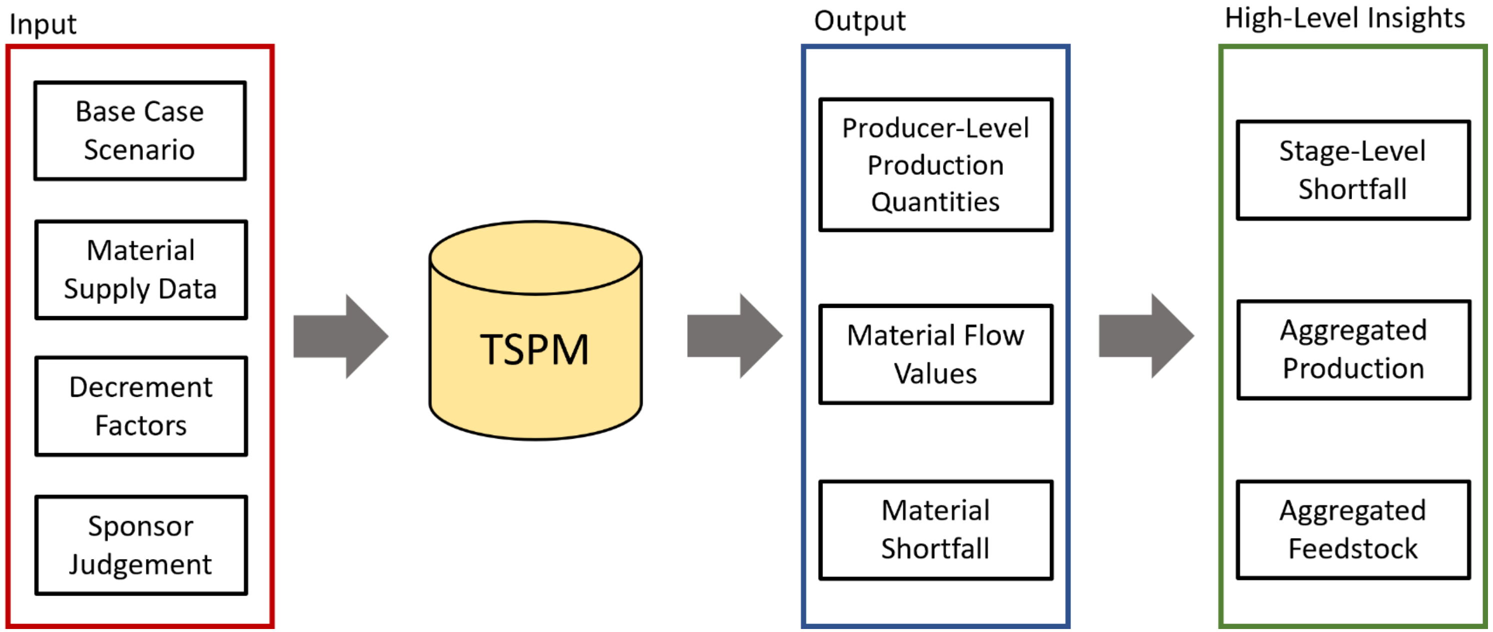
$$\text{Subject to:} \qquad \left. \begin{array}{l} \textit{Flow Feasibility} \\ \textit{Usability Considerations} \\ \textit{Production Capacity} \\ \textit{Demand Satisfied} \end{array} \right\} A\mathbf{x} \leq \mathbf{b}$$

Output:

- Production level of each producer during all planning years
- Units of material flow between producers
- Year-by-year supply to U.S. demand categories
- Year-by-year shortfalls of manufacture-ready material

High-Level Insights

- Total defense-usable and civilian-only-usable mined material feedstock by year
- Stage-level shortfalls; identification of bottlenecks in supply chain
  - Can inform domestic production efforts and determinations of which material form is best to stockpile
- Aggregated production quantities: domestic, foreign, defense-usable, and civilian-only-usable



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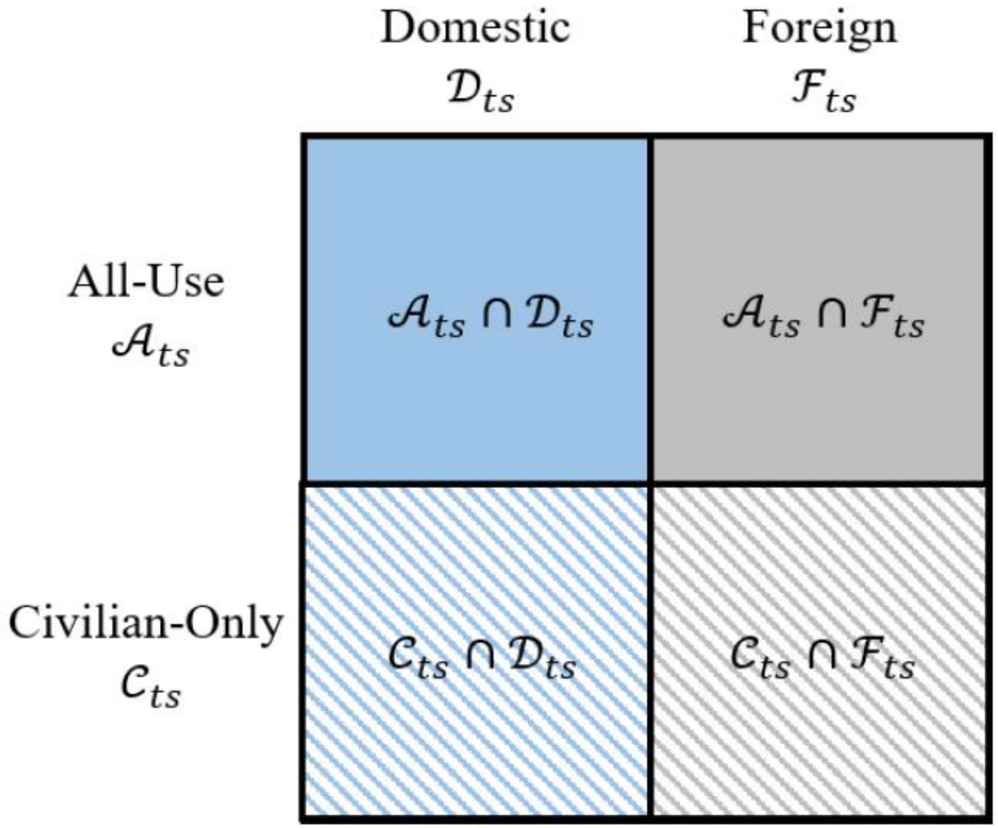
Notation Summary

Symbol	Description
	Producer Sets
$\mathcal{P}_{ts}$	Set of all stage-s producers during year t, indexed $1, \dots, m_s$
$\mathcal{A}_{ts}$	Set of stage-s all-usable producers during year t, indexed $1, \dots, k_s$ for $k_s \leq m_s$
$\mathcal{C}_{ts}$	Set of stage-s civilian-only-usable producers during year t, indexed $k_s + 1, \dots, m_s$
$\mathcal{D}_{ts}$	Set of domestic stage-s producers during year t
$\mathcal{F}_{ts}$	Set of foreign stage-s producers during year t
	Input Parameters
$Cits$	Production capacity of stage-s producer i in year t, given as units of material
$D_t^{\text{(def)}}$	U.S. defense demand in year t, given as units of material
$D_t^{\text{(civ)}}$	U.S. civilian demand in year t, given as units of material
K_{uv}	Arc cost on arc $(u, v) \in \mathcal{E}$. For more details on arc notation for specific arcs, see Table 1

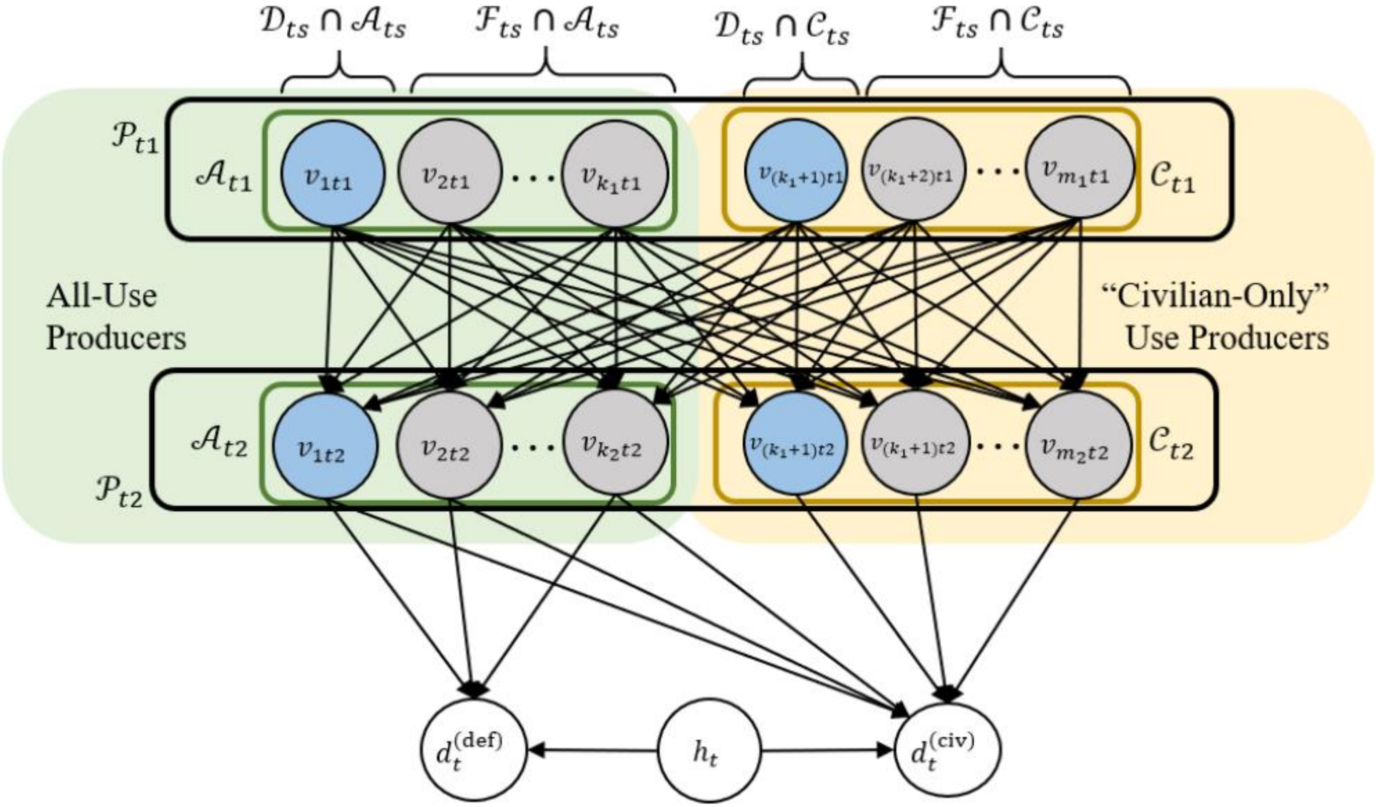
Classification of Stage-s Producers in Year t



Note: Producers fall into one of four categories based on usability and location (counterclockwise, from upper left): Domestic All-Usable, Domestic Civilian-Only-Usable, Foreign Civilian-Only Usable, and Foreign All-Usable.

Figure 2. Characterization of Stage-s Producers at Time t

Network Representation of Supply Chain with Decision Variables



Note: Here, we assume that there are $|\mathcal{A}_{ts}| = k_{ts}$ all-use producers, indexed $1, \ldots, k_{ts}$ and $|\mathcal{C}_{ts}| = m_{ts} - k_{ts}$ civilian-only-use producers, indexed $k_{ts} + 1, \ldots, m_{ts}$ in stage-s. Blue and gray nodes represent domestic and foreign producers, respectively.

Figure 3. Visualization of the Network Representation of the Supply Chain at a Fixed Time *t*

Decrement Factors

	Decrement Factors
$\hat{\beta}_{ijt}$	Shipping loss factor between stage-1 producer i and stage-2 producer j for material processed by producer i in year t
$\bar{\beta}_{it}$	Shipping loss factor between stage-2 producer i and the U.S. for material processed by producer i in year t
α_{its}	Ability factor of producer i in year t, computed as the product of multiple capacity reduction factors
δ_{its}	Willingness factor of stage-s producer i in year t, interpreted as the fraction of material not subject to delay
ρ_t	Market share factor for production in year t
γ_{it}	Wastage factor for stage-2 material processed by producer i in year t

Linear Program Decision Variables

Decision Variables	
$f_{itj\tau}$	Units of material flow from stage-1 producer $i \in \mathcal{P}_{t1}$ in year t to stage-2 producer $j \in \mathcal{P}_{\tau 2}$ in year τ
p_{its}	Units of material production from stage-s producer $i \in \mathcal{P}_{ts}$ in year t
$x_{it\tau}^{(\text{def})}$	Units of material processed by stage-2 producer i in year t flowing into U.S. defense demand node in year τ
$x_{it\tau}^{(\text{civ})}$	Units of material processed by stage-2 producer i in year t flowing into U.S. civilian demand node in year τ
$\sigma_t^{(\text{def})}$	Units of material flow from dummy supply node into U.S. defense demand node in year t
$\sigma_t^{(\text{civ})}$	Units of material flow from dummy supply node into U.S. civilian demand node in year t

Linear Programming Formulation (No Decrements)

$$
\begin{array}{llll}
\text{Minimize} & z(\mathbf{f}, \mathbf{x}, \boldsymbol{\sigma}, \mathbf{p}) & & (6)^{12} \\
\text{subject to:} & \displaystyle\sum_{t=t_0}^{T} \sum_{j \in \mathcal{P}_{t2}} f_{it_0jt} \leq p_{it_01} & \forall i \in \mathcal{P}_{t_01}, t_0 \in [T] & (7) \\
 & \displaystyle\sum_{t=t_0}^{T} \left[x_{it_0t}^{(\text{def})} + x_{it_0t}^{(\text{civ})}\right] \leq p_{it_02} & \forall i \in \mathcal{P}_{t_02}, t_0 \in [T] & (8) \\
 & \displaystyle p_{it_02} \leq \sum_{t=1}^{t_0} \sum_{j \in \mathcal{P}_{t1}} f_{jtit_0} & \forall i \in \mathcal{P}_{t_02}, t_0 \in [T] & (9) \\
 & p_{it_0s} \leq C_{it_0s} & \forall i \in \mathcal{P}_{ts}, s \in [2], t_0 \in [T] & (10) \\
 & \displaystyle\sum_{t=t_0}^{T} x_{it_0t}^{(\text{def})} \leq \sum_{t=1}^{t_0} \sum_{j \in \mathcal{A}_{t1}} f_{jtit_0} & \forall i \in \mathcal{A}_{t_02}, t_0 \in [T] & (11) \\
 & \displaystyle\sigma_{t_0}^{(\text{def})} + \sum_{t=1}^{t_0} \sum_{j \in \mathcal{A}_{t2}} x_{jtt_0}^{(\text{def})} = \mathrm{D}_{t_0}^{(\text{def})} & \forall t_0 \in [T] & (12) \\
 & \displaystyle\sigma_{t_0}^{(\text{civ})} + \sum_{t=1}^{t_0} \sum_{j \in \mathcal{P}_{t2}} x_{jtt_0}^{(\text{civ})} = \mathrm{D}_{t_0}^{(\text{civ})} & \forall t_0 \in [T] & (13) \\
 & & \mathbf{f}, \mathbf{x}, \boldsymbol{\sigma}, \mathbf{p} \geq 0 & (14)
\end{array}
$$

Linear Programming Formulation (With Decrements)

$$\text{Minimize} \quad z(\mathbf{f}, \mathbf{x}, \boldsymbol{\sigma}, \mathbf{p}) \tag{34}$$

$$\text{subject to:} \quad \sum_{t=t_0}^{T} \sum_{j \in \mathcal{P}_{t2}} f_{it_0jt} \le p_{it_01} \qquad \forall i \in \mathcal{P}_{t_01}, t_0 \in [T] \tag{35}$$

$$\sum_{j \in \mathcal{D}_{t_02}} f_{it_0jt_0} \le \delta_{it_01} \cdot p_{it_01} \qquad \forall i \in \mathcal{F}_{t_01}, t_0 \in [t] \tag{36}$$

$$\sum_{t=t_0}^{T} \left[x_{it_0t}^{(\text{def})} + x_{it_0t}^{(\text{civ})}\right] \le \gamma_{it_0} \cdot p_{it_02} \qquad \forall i \in \mathcal{P}_{t_02}, t_0 \in [T] \tag{37}$$

$$x_{it_0t_0}^{(\text{def})} + x_{it_0t_0}^{(\text{civ})} \le \gamma_{it_0} \cdot \delta_{it_02} \cdot p_{it_02} \qquad \forall i \in \mathcal{F}_{t_02}, t_0 \in [T] \tag{38}$$

$$p_{it_02} \le \sum_{t=1}^{t_0} \sum_{j \in \mathcal{P}_{t1}} \hat{\beta}_{jit} \cdot f_{jtit_0} \qquad \forall i \in \mathcal{P}_{t_02}, t_0 \in [T] \tag{39}$$

$$p_{it_0s} \le \alpha_{it_0s} \cdot C_{it_0s} \qquad \forall i \in \mathcal{P}_{ts}, s \in [2], t_0 \in [T] \tag{40}$$

$$\sum_{t=t_0}^{T} x_{it_0t}^{(\text{def})} \le \gamma_{it_0} \sum_{t=1}^{t_0} \sum_{j \in \mathcal{A}_{t1}} \hat{\beta}_{jit} \cdot f_{jtit_0} \qquad \forall i \in \mathcal{A}_{t_02}, t_0 \in [T] \tag{41}$$

$$\sigma_{t_0}^{(\text{def})} + \sum_{t=1}^{t_0} \left[\left(\sum_{k \in \mathcal{D}_{t2} \cap \mathcal{A}_{t2}} \bar{\beta}_{kt} \cdot x_{ktt_0}^{(\text{def})}\right) + \rho_t \left(\sum_{j \in \mathcal{F}_{t2} \cap \mathcal{A}_{t2}} \bar{\beta}_{jt} \cdot x_{jtt_0}^{(\text{def})}\right)\right] = \mathrm{D}_{t_0}^{(\text{def})} \qquad \forall t_0 \in [T] \tag{42}$$

$$\sigma_{t_0}^{(\text{civ})} + \sum_{t=1}^{t_0} \left[\left(\sum_{k \in \mathcal{D}_{t2}} \bar{\beta}_{kt} \cdot x_{ktt_0}^{(\text{civ})}\right) + \rho_t \left(\sum_{j \in \mathcal{F}_{t2}} \bar{\beta}_{jt} \cdot x_{jtt_0}^{(\text{civ})}\right)\right] = \mathrm{D}_{t_0}^{(\text{civ})} \qquad \forall t_0 \in [T] \tag{43}$$

$$\mathbf{f}, \mathbf{x}, \boldsymbol{\sigma}, \mathbf{p} \ge 0 \tag{44}$$

Formal Objective Function Expression

Table 1. Relationships Among LP Flow Variables, Network Arc Construction, and Arc Costs

LP Variable	Network Representation	Arc Cost	Description
$f_{itj\tau}$	$(v_{it1}, v_{j\tau2}) \in \mathcal{E}^1 \sqcup \mathcal{E}^4$	$K^f_{itj\tau}$	Material flow from stage-1 producer i in year t to stage-2 producer j in year $\tau \geq t$
$x^{(\text{def})}_{it\tau}$	$\left(v_{it2}, d^{(\text{def})}_\tau\right) \in \mathcal{E}^2 \sqcup \mathcal{E}^5$	$K^{x(\text{def})}_{it\tau}$	Material flow from stage-2 producer i in year t to U.S. defense demand node in year $\tau \geq t$
$x^{(\text{civ})}_{it\tau}$	$\left(v_{it2}, d^{(\text{civ})}_\tau\right) \in \mathcal{E}^2 \sqcup \mathcal{E}^5$	$K^{x(\text{civ})}_{it\tau}$	Material flow from stage-2 producer i in year t to U.S. civilian demand node in year $\tau \geq t$
$\sigma^{(\text{def})}_t$	$\left(h_t, d^{(\text{def})}_t\right) \in \mathcal{E}^3$	$K^{\sigma(\text{def})}_t$	Material flow from dummy supply node in year t to U.S. defense demand node in year t
$\sigma^{(\text{civ})}_t$	$\left(h_t, d^{(\text{civ})}_t\right) \in \mathcal{E}^3$	$K^{\sigma(\text{civ})}_t$	Material flow from dummy supply node in year t to U.S. civilian demand node in year t

$$z(\mathbf{f}, \mathbf{x}, \boldsymbol{\sigma}, \mathbf{p}) = \sum_{t=1}^{T} \Big[\left(K^{\sigma(\text{def})}_t \cdot \sigma^{(\text{def})}_t + K^{\sigma(\text{civ})}_t \cdot \sigma^{(\text{civ})}_t \right) + \sum_{\tau=t}^{T} \sum_{i \in \mathcal{P}_{t1}} \sum_{j \in \mathcal{P}_{\tau 2}} \left(K^f_{itj\tau} \cdot f_{itj\tau} \right)$$

$$+ \sum_{\tau=t}^{T} \sum_{i \in \mathcal{P}_{t2}} \left(K^{x(\text{def})}_t \cdot x^{(\text{def})}_{it\tau} + K^{x(\text{civ})}_t \cdot x^{(\text{civ})}_{it\tau} \right) \Big]$$

Image References

11A: Oat_Phawat/iStock via Getty Images. Lump of silver or platinum or rare earth minerals on black background.

11B: Dheo Tegar Pratama/iStock via Getty Images. Closeup of a bunch of metal pipes stacked on top of each other.

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14. ABSTRACT

Material supply chains are complex, global systems that drive the production of essential goods and services, from raw material extraction to manufacturing processes. As the U.S. becomes increasingly dependent on foreign production of these materials, ensuring security against potential shortages is paramount. To address this, IDA developed an analytical framework to help the Department of Defense (DoD) identify and mitigate potential material gaps. Central to this system is the Stockpile Sizing Module (SSM), which estimates material shortfalls during national emergencies. This talk will present an enhanced multi-commodity flow model that builds upon the SSM's network linear programming formulation, explicitly incorporating two key stages of supply—mining and refining—while tracking material throughout these stages and accounting for supply decrements. This improved framework provides more accurate shortfall calculations, allowing for the identification of critical bottlenecks and offering vital insights into how the U.S. can better prepare for future national security emergencies. The model strengthens IDA's existing analytic portfolio supporting DoD and the National Defense Stockpile, informing proactive defense and economic mobilization strategies by identifying critical risks and offering actionable recommendations to mitigate these vulnerabilities.

15. SUBJECT TERMS

Strategic and Critical Materials, Shortfall, Risk mitigation, Linear Optimization, Risk Assessment and Mitigation Framework for Strategic Materials, National Defense Industrial Strategy, Defense Logistics Agency, essential demand, safe supply, National Defense Stockpile, national security emergency planning case, stockpile acquisition priorities, material prioritization

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